Paper 1 — Abstraction: Structural Definition (Horizontal)

0. Abstract

This paper introduces a structural, substrate-agnostic definition of abstraction. Abstraction is defined as an upward operator that lifts coherent substructures from a base layer into a higher layer while preserving invariants and coherence, expanding degrees of freedom, and enabling new diagnostic capabilities. The paper identifies the structural primitives required for abstraction, distinguishes horizontal from vertical composition, and establishes the foundation for the mechanism of abstraction (Paper 2), the definition and mechanism of compression (Papers 3 and 4), and the duality between the two operators (Paper 5).

1. Introduction

1.1 The Problem: No Structural Definition of Abstraction Exists

Existing accounts of abstraction are cognitive, symbolic, representational, or reductive. They describe the *appearance* of abstraction rather than its *mechanism*. They collapse abstraction into a single layer, treat it as simplification or generalization, and fail to describe the structural transformation that occurs when a system moves from one layer to another. As a result, no substrate-agnostic definition of abstraction currently exists.

1.2 The Goal of This Paper

This paper defines abstraction as a structural operator. The focus is on **horizontal abstraction**, a single upward transition from a base layer to a higher layer. Vertical composition is introduced but not developed. The aim is to establish the primitives, geometry, and essential properties of the upward operator.

1.3 Scope and Boundaries

This paper defines abstraction and the structural conditions that make it possible. Paper 2 will describe the mechanism of abstraction. Papers 3 and 4 will define and develop compression. Paper 5 will establish the duality between abstraction and compression. Vertical composition is acknowledged but deferred.

2. Why Existing Definitions Fail

2.1 Symbolic and Cognitive Definitions

Symbolic and cognitive accounts treat abstraction as categorization, concept formation, or mental generalization. These definitions rely on human-specific metaphors and do not generalize across substrates.

2.2 Representational Definitions (Machine Learning)

Machine learning definitions treat abstraction as feature extraction or latent representation. These describe outputs rather than operators and conflate abstraction with representation.

2.3 Simplification and Reduction Definitions

Many definitions treat abstraction as simplification or information reduction. These are reductive operations aligned with compression, not abstraction. They do not describe a cross-layer transformation.

2.4 Systems Theory Definitions

Coarse-graining and multi-scale modeling capture some structural aspects but do not define abstraction as an operator. They conflate aggregation with upward transformation.

2.5 Summary of Failure Modes

Existing definitions are single-layer, reductive, or representational. None describe abstraction as a structural operator acting across layers. The common definition of abstraction captures a reductive effect that often accompanies certain cognitive shortcuts. This effect is directionally aligned with compression, not abstraction, and lacks the structural properties required for true compression. Because it does not operate across layers, it cannot serve as a definition of abstraction. In this paper, abstraction is defined as an upward structural operator, distinct from both simplification and compression.

3. Structural Primitives Required for Abstraction

Abstraction requires a minimal set of structural primitives. These primitives are not cognitive or symbolic; they are architectural conditions that make an upward operator possible.

3.1 Layers

A system must contain at least two coherent layers. Abstraction is defined relative to these layers.

3.2 Structure

Abstraction operates on coherent substructures, not arbitrary data.

3.3 Invariants

Certain properties must remain stable across the layer transition. These invariants anchor identity.

3.4 Coherence

The internal relationships and constraints of the structure must remain intact.

3.5 Degrees of Freedom

The higher layer must offer more expressive or operational possibilities than the lower layer.

3.6 Diagnostics

The higher layer must support analyses or transformations that the lower layer cannot.

3.7 Closure Under Composition

Horizontal composition: different substructures can be independently lifted. Vertical composition: the output of abstraction can serve as input to further abstraction (introduced only).

4. Horizontal and Vertical Composition

Abstraction is compositional. It can be applied to different substructures or repeatedly to the same substructure.

4.1 Horizontal Composition (Scope of This Paper)

Horizontal composition lifts different substructures into a higher layer. Each undergoes a single upward transition. This is the form of abstraction defined in this paper.

4.2 Vertical Composition (Introduced Only)

Vertical composition recursively abstracts the same substructure across multiple layers. It requires additional primitives and is not developed here.

4.3 Why This Paper Focuses on Horizontal Composition

Horizontal abstraction is the most generalizable form and establishes the foundation for the mechanism of abstraction, compression, and duality.

5. Formal Definition of Abstraction (Horizontal)

Abstraction is an upward structural operator. It lifts a coherent substructure from a base layer into a higher layer while preserving invariants and coherence, expanding degrees of freedom, and enabling new diagnostic capabilities.

5.1 Definition

Abstraction is an upward structural operator that extracts a coherent substructure from a base layer, preserves its invariants, and re-expresses it in a higher layer with expanded degrees of freedom and new diagnostic capabilities.

5.2 Structural Mapping Across Layers

Abstraction relocates a structure from one layer to another. The higher layer must preserve invariants and coherence while offering expanded expressive and diagnostic capabilities. The upward transition is not reductive; it is a re-expression in a more capable layer.

5.3 Essential Properties

- Upward directionality
- Invariant preservation
- Coherence preservation
- Degree-of-freedom expansion
- Diagnostic expansion
- Closure under horizontal composition
- Closure under vertical composition (introduced only)

5.4 What Abstraction Is Not

Abstraction is not simplification, generalization, categorization, information reduction, symbolic representation, or cognitive filtering. These reductive operations lack the structural properties required for a cross-layer operator.

5.5 Summary

Abstraction lifts, preserves, expands, and enables. This definition provides the foundation for the mechanism of abstraction and the duality between abstraction and compression.

6. Structural Examples of Horizontal Abstraction

6.1 Neural Systems

Local spatial patterns in pixel data are lifted into a higher representational layer, preserving structure while gaining expressive and diagnostic capabilities.

6.2 Biological Regulatory Networks

A regulatory motif is lifted from molecular interactions into a network-motif layer, enabling functional and dynamical analysis.

6.3 Cognitive Systems (Non-Symbolic)

A relational pattern in sensory data is lifted into a higher cognitive layer, enabling prediction and manipulation.

6.4 Distributed Systems

A communication pattern is lifted from protocol primitives into a protocol layer, enabling verification and optimization.

6.5 Mathematical Systems

A geometric configuration is lifted into a similarity-class layer, enabling transformation and equivalence reasoning.

7. Implications for Compression and Duality

7.1 Why Compression Requires Its Own Definition

A complete account of layered systems requires a downward operator that returns structures to lower layers while preserving invariants and coherence. This operator is compression. It is not simplification and must be defined separately.

7.2 Why Duality Cannot Be Introduced Yet

Duality requires both operators to be fully defined. Introducing it prematurely would collapse the membrane.

7.3 How This Paper Prepares for the Next Papers

This paper establishes the primitives and geometry needed for the mechanism of abstraction (Paper 2), the definition and mechanism of compression (Papers 3 and 4), and the duality between the two operators (Paper 5).

8. Conclusion

This paper introduced a structural definition of abstraction as an upward operator that lifts coherent substructures into higher layers while preserving invariants and coherence, expanding degrees of freedom, and enabling new diagnostics. Horizontal composition was defined as the accessible form of abstraction; vertical composition was acknowledged but deferred. This definition establishes the foundation for the mechanism of abstraction, the definition and mechanism of compression, and the duality between the two operators. It provides the first step in a coherent framework for understanding cross-layer transformations in complex systems.